On the morpho-semantic puzzle of superlative modifiers

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1 Introduction

Superlative modifiers (SMs) such as English at least and at most have posed a longstanding and intriguing morpho-semantic puzzle: Why do SMs morphologically involve a quantity adjective and the superlative morpheme? What is the role of quantity adjective and superlative morpheme inside SMs? How are these morphological pieces connected with the semantics of SMs? Below, the relevant facts in English are illustrated in (1) and (2), where the same morphological components least and most are involved in both SMs and QSs.  

(1) Superlative modifiers (SMs)
   a. John bought at least [three] apples.
   b. John bought at most [three] apples.

(2) Quantity superlatives (QSs)
   a. John drank the least water.
   b. John climbed the most mountains.

In this respect, Chinese makes the situation even more puzzling; in particular, the expressions for SMs and QSs are twins on the morphological makeup: exactly the same expressions (morphologically consisting of a quantity adjective and superlative morpheme) zui-duo and zui-shao are used as SMs and QSs.  

(3) a. Liubei mai-le zui-duo (ke) pinguo.
     Liubei buy-ASP SUP-many CL apple
     ‘Liubei bought more apples than anyone else did.’

   Liubei buy-ASP SUP-many three-CL apple
   SMs

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1 It has been established in Hackl (2000, 2009) that quantity superlatives, unlike quality superlatives, have only the relative/comparative reading, as shown in (i) and (ii).

(i) Adam climbed the highest mountain. ambiguous
   Absolute reading: Adam climbed Mount Everest.
   Relative reading: Adam climbed a higher mountain than anyone else did.

(ii) Adam climbed the most mountains. unambiguous
   Relative reading: Adam climbed more mountains than anyone else did.
‘Liubei bought at most three apples.’

(4) a. Liubei mai-le zui-shao (ke) pinguo. Qs
   [Liubei buy-ASP SUP-little CL apple]
   ‘Liubei bought fewer apples than anyone else did.’

b. Liubei mai-le zui-shao [san]F-ke pinguo. Sm
   Liubei buy-ASP SUP-little three-CL apple
   ‘Liubei bought at least three apples.’

In (3a), zuiduo conveys the relative reading of superlatives: the quantity of apples that Liubei bought is more than the quantity of apples that any other relevant individual did. In contrast, in (3b), zuiduo conveys an upper bound on the number of apples that Liubei bought. The same contrast holds for (4) with the expression zuishao, though with a reversed polarity.

This paper takes Chinese SMs as a case study and presents a decompositional analysis. The rest of this paper proceeds as follows. Section 2 introduces three crucial properties of SMs that this paper is concerned with. Section 3 spells out the core ingredients of my proposal and presents a decompositional analysis of Chinese SMs. Section 4 concludes the paper.

2 Basic facts

This section introduces three empirical properties of SMs (some of them have been observed in English, see e.g., Krifka 1999, Coppock and Brochhagen 2013): (a) they are focus-sensitive; (b) they are compatible with various types of scales and respect the monotonicity constraint; (c) they make the prejacent the lower/upper bound among the set of focus alternatives. First of all, Chinese SMs are focus-sensitive: the semantic contributions of zuiduo/zuishao depend on its focus associate and different positions of the associate lead to truth-conditional differences. Consider the contexts in (5) and (6), and the utterances in (7) and (8).

(5) Context A: a contextual ranking: cherries > apples > bananas
   What did Liubei buy for our plan tonight?

(6) Context B: a contextual ranking: make dinner > buy apples > boil water
   What did Liubei do for our plan tonight?

(7) Liubei zuiduo/ zuishao mai-le [pinguo]F.
   Liubei SUP-many SUP-little buy-ASP apple
   ‘Liubei at most/ at least bought apples.’

(8) Liubei zuiduo/ zuishao [mai-le pinguo]F.
   Liubei SUP-many SUP-little buy-ASP apple
   ‘Liubei at most/ at least bought apples.’

Due to different positions of focus associate, (7) and (8) are truth-conditionally different. In (7), a lower bound (in the case of zuishao) and an upper bound (in the case of zuiduo) is imposed on what Liubei has bought. In contrast, the relevant
Bounding property is placed on what Liubei has done in (8). Therefore, (7) is a felicitous continuation to the question in context A, but (8) is not. Conversely, (8) is a felicitous continuation to the question in context B, but (7) is not.

Second, Chinese SMs are compatible with various scales, as shown below.²

(9) **Numerical Scales** (a contextual ranking: 4 \(>\) 3 \(>\) 2)

Liubei zui-duo/ zui-shao xie-le [san]_f-ben-xiaoshuo.
Linbei SUP-many SUP-little write-ASP three-CL-novel
‘Liubei at most/ at least wrote three novels.’

(10) **Plurality Scales** (a contextual ranking: \(a\oplus b\oplus c > a\oplus b > a\))

Liubei zui-duo/ zui-shao guyong-le [Adam he Bill]_f.
Liubei SUP-many SUP-little hire-ASP Adam and Bill
‘Liubei at most/ at least hired Adam and Bill.’

(11) **Lexical Scales** (a contextual ranking: gold \(>\) silver \(>\) bronze)

Liubei zui-duo/ zui-shao na-le [yin]_f-pai.
Liubei SUP-many SUP-little take-ASP silver-medal
‘Liubei at most/ at least got a silver medal.’

(12) **Pragmatic Scales** (a contextual ranking: cherries \(>\) apples \(>\) bananas)

Liubei zui-duo/ zui-shao mai-le [pingguo]_f.
Liubei SUP-many SUP-little buy-ASP apple
‘Liubei at most/ at least bought apples.’

Note that the numerical scale and plurality scale are based on semantic strength (i.e., entailment relation). Therefore, writing four novels entails writing three

² In Chinese, when *zuiduo/ zuishao* occur with proper names or quantifiers in a prenominal position, the sentences are reported to be degraded for some native speakers.

(i) ? Liubei guyong-le zui-duo/ zui-shao [Adam he Bill]_f.
Liubei hire-ASP SUP-many SUP-little Adam and Bill
‘Liubei hired at most/ at least Adam and Bill.’

(ii) ?? Liubei yaoqing-le zui-duo/ zui-shao [yi-xie-xueshen]_f.
Liubei invite-ASP SUP-many SUP-little one-CL-student
‘Liubei invited at most/ at least some students.’

However, the sentences become perfect when *zuiduo/ zuishao* occur in a preverbal position.

(iii) Liubei zui-duo/ zui-shao guyong-le [Adam he Bill]_f.
Liubei SUP-many SUP-little hire-ASP Adam and Bill
‘Liubei at most/ at least hired Adam and Bill.’

(iv) Liubei zui-duo/ zui-shao yaoqing-le [yi-xie-xueshen]_f.
Liubei SUP-many SUP-little invite-ASP one-CL-student
‘Liubei at most/ at least invited some students.’

At this moment, I have nothing interesting to say about the contrast, other than suggesting that the contrast may be reduced to certain syntactic competition or incompatibility between the position of prenominal *zuiduo/ zuishao* and that of proper names/ quantifiers in Chinese. I leave this line of research for another occasion.
novels, and hiring Adam and Bill entails hiring Adam and hiring Bill. In contrast, the lexical scale and pragmatic scale are based on pragmatic strength (i.e., non-entailment relation). Thus, winning a gold medal does not entail winning a silver medal, and buying apples does not entail buying bananas.

Moreover, the ranking of the alternatives seems to respect the monotonicity property (cf. Schwarzschild 2006). By manipulating the context, it seems easy enough to reverse the ordering between the alternatives in the case of pragmatic scales. In contrast, however, it does not seem possible to reverse the ordering in the case of numerical scales or plurality scales, even with some contextual effort.

(13) Context: Caocao, Liubei, Sunquan are planning to buy some fruit for their party tonight. There are three types of fruit available to them: cherries, apples and bananas. However, they are poor and do not have enough money to buy everything. For them, bananas are the optimal because they are the cheapest; apples are less optimal but acceptable because they are still cheaper than cherries.

The contextual ranking (in terms of price): bananas > apples > cherries

(14) Context: Liubei is planning to hire some people. There are three applicants in the discourse: Adam, Bill and Chris. But the budget is limited. If three people are all hired, Liubei need to pay a great amount of money for their salary. If only Adam and Bill are hired, the situation is better, but Liubei still pays more than he does in hiring only Adam. The best situation for Liubei is simply to hire just one person while getting all the work done.

The intended contextual ranking:
only Adam > only Adam & Bill > only Adam & Bill & Chris

Under the context (13), the utterance with zuishao in (12) is understood to convey that Liubei bought apples or bananas (given the contextual ranking: bananas > apples > cherries). This means that the original ranking (cherries > apples > bananas) in (12) is now reversed. In contrast, the utterance with zuishao in (10) cannot be understood to be that Liubei hired only Adam and Bill, or hired only Adam, even with the contextual massage in (14). This indicates that the original ranking (adam & bill & chris > adam & bill > adam) in (10) cannot be reversed. The same observation applies to the numeral scale. I leave it for readers to verify the case of numerical scales. These facts suggest that there is an intrinsic discrepancy between scales based on semantic strength and those based on pragmatic strength.

Finally, under sentences with SMs, the prejacent is made as the lower/upper bound among the set of focus alternatives, as illustrated by English in (15).

3 For lexical scales, although context manipulations are not impossible, they are harder because the ordering is based on our common world knowledge. Instances of lexical scale are gold medal > silver medal > bronze medal, and full professor > associate professor > assistant professor.
The empirical facts to be captured in this paper are summarized in (16).

(16)  a. **The morpho-semantic puzzle:** The same expressions *zuíduo* and *zuishao* are used in superlative modifiers and quantity superlatives.

b. **Focus-sensitivity:** The semantic contribution of *zuíduo* and *zuishao* depends on the position of their focus associate.

c. **Scale types and their discrepancy:** *Zuíduo* and *Zuishao* are compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between the alternatives cannot be reversed in numerical scales and plurality scales.

d. **The bounding property:** Under sentences with SMs, the prejacent is set up as the lower/upper bound among the set of focus alternatives.

The next section spells out my decompositional analysis of Chinese SMs, capturing those empirical facts discussed above.

### 3 The proposal

The section proceeds as follows. Section 3.1 shows that Chinese SMs can be structurally decomposed into three major components: a quantity adjective (Q-adjective), a superlative component SupP and a covert existential operator E-OP. Section 3.2 illustrates the compositions of SMs and section 3.3 the compositions of QSs. Specifically, exactly the same superlative component SupP is involved in both constructions; this captures the fact that SMs and QSs are twins in Chinese.

Below, I briefly introduce two core theoretical assumptions in this paper. First, I assume Rooth (1992)’s focus semantics. Briefly put, every expression \( \varphi \) has an ordinary semantic value and a focus semantic value. For an unfocused constituent, its focus semantic value is a singleton set containing the ordinary value of that expression. For a focused constituent, its focus semantic value is a set of alternatives: a set of objects that have the same semantic type as the focused constituent. The set of alternatives induced by focus is computed recursively (essentially as in Rooth 1992). Furthermore, the semantic contribution of a focus-sensitive operator depends on the focus semantic value of its sister. The set of
focus alternatives projects until they meet the focus operator where they are interpreted by a squiggle operator ~ and restricted by a contextual variable C. The definition of ~ in (17) is drawn from Rooth (1996: 20).

(17) Where φ is a syntactic phrase and C is a syntactically covert semantic variable, $φ \sim C$ introduces the presupposition that C is a subset of $⟦φ⟧^f$ containing $⟦φ⟧^o$ and at least one another element.

Second, I assume Bobalijk (2012)’s Containment Hypothesis, as in (18).

(18) Containment Hypothesis (Bobalijk 2012)
     A superlative structurally embeds a comparative.

The next section presents a decomposition of Chinese SMs zuiduo and zuishao.

3.1 Decomposing superlative modifiers (SMs)

To begin with, recall that Chinese SMs can be structurally decomposed into three major components: a quantity adjective (Q-adjective), a superlative component SupP and a covert existential operator E-OP. Assuming Bobalijk (2012)’s Containment Hypothesis, I propose that in Chinese SMs, the superlative construction structurally embeds a comparative construction, as illustrated in (19). Note that CompP represents the embedded covert comparative construction.

(19) $[\text{SupP } \text{zui} [\text{CompP } \text{Comp}^0 [\text{AdjP } \text{duo}]])$                      SupP

Regarding the semantic details, let’s start in a bottom-up fashion. For the quantity adjective duo, I propose that it encodes a measure function $μ_c$, which maps the elements induced by focus to their corresponding positions along a contextually given dimension, as defined in (20). Note that $η$ is a meta-variable, intended to capture the fact that the semantic type of focused elements may vary.

(20) $⟦\text{duo}⟧^c = \lambda α.μ_c(α)$                                                $<η, d>$

Moreover, I propose that in the case of SMs, the measure function $μ_c$ respects the monotonicity constraint, but crucially not restricted to it (cf. Wellwood 2014, 2015). This has two consequences immediately. First, Chinese SM zuiduo (and their cross-linguistic counterparts) can apply to the alternatives whose domain is not structured (e.g., by the part-of relation), as in the case of lexical scale and pragmatic scale. Second, when the domain of the alternatives is structured, as in the case of numerical scale and plurality scale, the structure-preserving mapping guarantees that the output ranking between the alternatives cannot be altered. The definition of monotonicity is offered below.

(21) Monotonicity
     A measure function $μ: D_{\text{Part}} \to D_{\text{Deg}}$ is monotonic iff
     for all $α, β ∈ D_{\text{Part}}$, if $α \sim_{\text{Part}} β$, then $μ(α) <_{\text{Deg}} μ(β)$.

Given the present analysis, the discrepancy between different scale types is now derived: When the input focus alternatives have their own internal structure (e.g.,
a partial ordering or a total ordering), through a structure-preserving mapping, the output ranking between the alternatives cannot be altered despite contextual effort (e.g., numerical scale and plurality scale). In contrast, when the set of focus alternatives is NOT structured by entailment relation or part-of relation, the output ranking between the alternatives is subject to contextual factors and thus is not constant across contexts (e.g., lexical scale and pragmatic scale).

Next, the comparative morpheme Comp$^+$ (cf. English -er) takes the Q-adjective duo as its first argument and returns a comparison relation between the alternatives, as in (22). The semantics of the superlative morpheme zui offered in (23) is like the entry traditionally assigned to English –est (e.g., Heim 1999), except for the additional comparison relation and its type-flexibility. The semantics of SupP is obtained, as in (24), when we put all the pieces together.

(22) $\llbracket\text{Comp}^+P\rrbracket = \lambda\alpha\beta.\mu_c(\alpha) > \mu_c(\beta)$

(23) $\llbracket\text{zui}\rrbracket = \lambda\text{COM} <\eta, <\eta, >\cdot \forall\beta[\beta\in C \land \beta \neq \alpha \rightarrow \text{COM}(\alpha, \beta)]$

(24) $\llbracket\text{SupP zui} [\text{Comp}^+P \text{Comp}^+] [\text{AdjP duo}]\rrbracket = \lambda\text{C} <\eta, >\cdot \forall\beta[\beta\in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]$ SupP

Finally, a covert existential operator (propositional version) is provided below.

(25) $\llbracket\text{E-OP}\rrbracket = \lambda\text{SUP} <\eta, >\cdot \forall\text{C} <\eta, >\cdot \exists\gamma[\gamma\in C \land \gamma_w \land \text{SUP} (C, c)]$

With the above building blocks in place, the internal structure of zuiduo is shown in (26) and the semantics of zuiduo (a propositional version) is presented in (27).

(26) The internal structure of Chinese SM zuiduo ‘at most’ at LF

(27) $\llbracket\text{zuiduo}(C)\rrbracket = \lambda\text{zuiduo} <\eta, >\cdot \exists\gamma[\gamma\in C \land \gamma_w \land \forall\beta[\beta\in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]]$
negation contributed by adjectives with negative polarity (glossed as LITTLE in their analyses) and the comparative morpheme –er.

(28) NEG-Comp$^+$ is reanalyzed as Comp$^-$ (Heim 2006a,b; Büring 2007)

The semantics of Comp$^-$ (cf. English less) is defined in (29), taking Q-adjective duo as its first argument and returning a comparison relation between the alternatives along a contextually-valued scale. With the same meaning pieces of duo and zuí, the semantics of SupP involved in zuíshào ‘at least’ is offered in (30).

(29) $[\text{Comp } P]^c = \lambda \alpha \lambda \beta \mu_c(\alpha) < \mu_c(\beta)$

(30) $[[\text{SupP } zuí } [\text{Comp}^- \text{Comp } [\lambda \text{AdjP } \text{duo}]]]^c = \lambda C_\beta. \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$

Finally, the semantics of zuíshào (propositional version) is obtained in (31), after combining with E-OP. 4 The overall internal structure of zuíshào is shown in (32).

(31) $[[\text{zuíshào}(C)]^c = \lambda C_\gamma. \exists \gamma \in C \land \gamma_w \land \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$

(32) The internal structure of Chinese SM zuíshào ‘at least’ at LF

Before leaving this section, let me briefly highlight several important features of the current decompositional analysis of SMs: (a) Q-adjectives play a crucial role in encoding a measure function mapping the alternatives to their

4 To capture the fact that SMs can be syntactically adjoined to constituents of non-propositional meanings, I assume that Chinese SMs also have the following non-propositional lexical entries, which are derived by type-shifting (e.g., the Geach rule and the backward Geach rule).

(i) a. A non-propositional version (by the Geach rule)

$[[\text{zuíduō}(C)]^c = \lambda C_\gamma. \lambda P. \exists \gamma \in C \land \gamma_w \land \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]$

b. A non-propositional version (by the backward Geach rule)

$[[\text{zuíduō}(C)]^c = \lambda C_\gamma. \lambda P. \exists \gamma \in C \land \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) > \mu_c(\beta)]$

(ii) a. A non-propositional version (by the Geach rule)

$[[\text{zuí}(C)]^c = \lambda C_\gamma. \lambda P. \exists \gamma \in C \land \gamma_w \land \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$

b. A non-propositional version (by the backward Geach rule)

$[[\text{zuí}(C)]^c = \lambda C_\gamma. \lambda P. \exists \gamma \in C \land \forall \beta \in C \land \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$

For reasons of space and purposes of illustration, in this paper, I will only focus on cases where Chinese SMs are syntactically adjoined to constituents of propositional meanings.
corresponding positions ordered along a contextually-valued scale; this provides the foundation of the scalarity of SMs; (b) the superlative component SupP has dual roles: it not only serves as a domain restrictor of SMs but also crucially introduces the scalarity of SMs, which is a comparison relation between the prejacent and its alternatives; (c) the internal structure of both zuiduo and zuishao contains a superlative component, which in turn structurally embeds a comparative (Containment Hypothesis); (d) the existential operator E-OP in Chinese is covert, which implicates that it might be overt in other languages.

The next two sections illustrate how the compositions of SMs and QSs in Chinese are computed and how those empirical properties of SMs are captured.

3.2 Compositions: superlative modifiers (SMs)

Let’s first consider zuiduo ‘at most’. The relevant sentence is presented in (33), along with its LF in (34). The computation is illustrated in (35).

(33) Liubei zuiduo shi yi-wei [fu]_F -jiaoshou.
    Liubei SUP-much be one-CL associate-professor
    ‘At most, Liubei is an associate professor.’

(34) LF: [zuiduo(C)] [\forall v Liubei is an [associate]F professor] ~C]

(35) a. \([zuiduo(C)]^{w,c} = \lambda x.\exists y (y \in C \land y.w \land \forall \beta (\beta \in C \land \beta \neq x. \rightarrow \mu_c(x) > \mu_c(\beta)))]
   b. a.~C is defined iff \([a]^{0} \in C \land \exists a' (a' \neq a \land [a']^{0} \in C) \land C \subseteq [a]^{f}
   c. \([34]^{w,c} = 1 \iff \exists y (y \in C \land y.w \land \forall \beta (\beta \in C \land \beta \neq \lambda w.\text{Liubei is an associate professor in } w \rightarrow \mu_c(\lambda w.\text{Liubei is an associate professor in } w) > \mu_c(\beta)))]

Because of the presuppositions introduced by the ~ squiggle operator (Rooth 1992), the prejacent is one element in the domain C. Furthermore, because of the domain restrictor SupP, all the elements non-identical to the prejacent are ranked below the prejacent; this amounts to removing the lower alternatives from the domain C. Taken together; the domain C further restricted by SupP is now a set consisting of only the pejacent and its lower alternatives. According to (35), (33) is judged true if and only if there is one element in the domain (i.e., a set consisting of the prejacent and its lower alternatives) such that the element is true. This seems intuitively correct. Consider an academic ranking like full professor > associate professor > assistant professor, the sentence (33) is true only if Liubei is an associate professor or an assistant professor.  

It is well-known that at most and other negative quantifiers such as less than are compatible with the zero element; for instance, the sentence at most three students left is judged true even if no student(s) left. Under the present analysis, the question boils down to whether the zero element/individual is actually included in the domain; Readers are referred to Buccola and Spector (2016), and Bylinina and Nouwen (2018) for recent discussions on the zero element/individual.
Next, let’s turn to *zuishao* ‘at least’. The relevant sentence is shown in (36), along with its LF in (37). The semantic computation is illustrated in (38).

    Liubei SUP-little be one-CL associate-professor
    ‘Liubei is at least an associate professor.’

(37) LF: [[vp *zuishao*(C) [vp Liubei is an [associate]_F professor] ~C]]

(38) a. \[\text{\{zuishao}(C)\}]^{w,c} = \lambda\alpha_{<\lambda}. \exists \gamma[\gamma \in C \wedge \gamma_w \wedge \forall \beta[\beta \in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]

b. \(\alpha ~C\) is defined iff \([[\alpha]^o \in C \wedge \exists \alpha'[\alpha' \neq \alpha \wedge [[\alpha']]^o \in C] \wedge C \subseteq [[\alpha]^f]

c. \([[37]]^{w,c} = 1 \text{ iff } \exists \gamma[\gamma \in C \wedge \gamma_w \wedge \forall \beta[\beta \in C \wedge \beta \neq \lambda_w . \text{Liubei is an associate professor in } w \rightarrow \mu_c(\lambda_w . \text{Liubei is an associate professor in } w) < \mu_c(\beta)]]

Because of the presuppositions introduced by the ~ squiggle operator (Rooth 1992), the prejacent is one element in the domain \(C\). Furthermore, because of the domain restrictor SupP, all the elements non-identical to the prejacent are ranked above the prejacent. Put differently, the domain restrictor SupP removes the lower alternatives from the domain \(C\). Taken together; the domain \(C\) further restricted by SupP now denotes a set consisting of the pejacen t and its higher alternatives. According to (38), (36) is judged true if and only if there is one element in the domain (i.e., in the set consisting of the prejacent and its higher alternatives) such that the element is true.

At this point, it is worth noting that excluding the higher alternatives has a different pragmatic effect than excluding the lower alternatives, as illustrated by the contrast in (39) between *at most* and *at least* in English.

(39) a. This department store has **at least** 30% discount.

b. This department store has **at most** 30% discount.

Let us assume that the speaker is being cooperative and maximally informative. In the use of *at most*/ *zuiduo*, the speaker could have uttered one of the higher alternatives (e.g., 40%), but she didn’t. This leads to an inference that the speaker didn’t commit herself to the truth of the higher alternatives, but to the truth of the prejacent (via maxima of quality), given that the higher alternatives are more informative than the prejacent. In contrast, in the use of *at least*/ *zuishao*, the speaker have chosen the prejacent (rather than one of its lower alternatives, say, 20%) and she has committed to its truth presumably because the prejacent is more informative then any of its lower alternatives (via maxim of quantity). On this view, the negative flavor of *at most*/ *zuiduo* (contrast with *at least*/ *zuishao*), as shown in (39), may not come from the semantics, but from certain pragmatic reasoning on the choice of the prejacent under the speaker’s assertion (see also Krifka 1995a for a similar pragmatic account on the asymmetry between excluding higher and lower alternatives in the empirical domain of polarity items).
We have seen how utterances with *zuiduo* and *zuishao* are computed. Recall that SMs has three important semantic properties: (a) focus-sensitivity; (b) the compatibility with various scales and the discrepancy between scales of semantic strength and those of pragmatic strength; (c) the bounding property. Below, I elaborate how these empirical facts are captured under the present analysis of SMs. First, the focus-sensitivity follows because focus induces a set of elements, serving as the input to the domain of the measure function. Second, the compatibility with various scales follows because the dimension of measurement is contextually-given; moreover, the discrepancy between scales of semantic strength and those of pragmatic strength follows because the mapping from the set of focus alternatives to their corresponding positions along a contextually-given scale is structure-preserving (i.e., monotonicity; see (21)). Third, the bounding property follows from focus presuppositions (i.e., the prejacent is included) and the contribution of SupP (i.e., the higher/lower alternatives are included).

The next section illustrates how sentences with quantity superlatives (QSs) are computed and explains why SMs and QSs in Chinese are twins in their morphological makeup. For reasons of space and purposes of illustration, I will focus on the computation of QSs in the nominal domain.

### 3.3 Compositions: Quantity Superlatives (QSs)

Let’s first consider the case of *zuiduo*. The relevant sentence is presented in (40), along with its LF in (41). Notice that the syntactic chunk \([zui\text{-Comp}\,^+\,duo]\)-C is the familiar superlative component SupP we have seen in Chinese SMs above.

(40) QSs in Nominal Domain

Liubei mai-le zui-duo (ke) pinguo.

Liubei buy-ASP SUP-many CL apple

‘Liubei bought more apples than anyone else did.’

(41) \([\text{SupP}\,zui\text{-Comp}\,^+\,duo]\)-C \(\lambda d\,_{IP\,\exists\,\text{AdjP}\,d\text{-[M-OP}_2\text{ apples]}}\,_{\text{C}}\)

Three remarks are in order. First, following Rett (2008, 2014), I assume the existence of M-OP defined in (42), of type \(\leq \eta, \leq t\,\rangle\). Note that under Rett’s original formulation, M-OP relates individuals \(x\) in the extension of some predicate \(P\) to their degrees along some contextually valued dimension. Here, I generalize her idea in a way that M-OP can be applied to both nominal domain and verbal domain. More specifically, in the case of QSs, \(\alpha\) ranges over events and individuals, thus \(\eta\) could be of type \(\leq \nu\) or \(\leq \epsilon\).

(42) \([\text{M-OP}_2]\,^c = \lambda P\,_{\leq \eta}, \lambda d\,_{\leq \eta}, \lambda \alpha_{\leq \eta}, |P(\alpha) \wedge \mu_2(\alpha) \geq d|\)

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6 This is intended to capture the fact that *zuiduo* and *zuishao* as QSs are well-formed in the verbal domain. Although I do not discuss the case of QSs in the verbal domain here, the computation would be basically similar to that in the nominal domain, except for the type difference.
Second, to simplify the derivation, I abstract away the denotation of classifiers and kinds (e.g., Krifka 1995b, Chierchia 1998) and assume that existential closure closes the individual variable. Assuming exactly the same meaning pieces of quantity adjective *duo*, Comp^{+}P and the superlative morpheme *zui* (see section 3.2 above), we obtain our familiar SupP again, as shown in (43).  

(43) \[ [[_{\text{SupP}} \text{zui-Comp}^{+} \text{duo}] \ldots C]^{c} \]
\[ = \lambda C_{<\ldots, \ldots \ldots, \ldots >} \lambda \alpha_{<\ldots, \ldots \ldots, \ldots >} \cdot \forall \beta[\beta \in C \land \beta \neq \alpha \rightarrow \mu_{1}(\alpha) > \mu_{1}(\beta)] \]
\[ = (24) \]

Third, for illustrative purposes, I assume with Heim (1999) that the relative reading of superlatives is derived by the movement of the superlative phrase.

The computation of *zuiduo* as QSs in the nominal domain is shown in (44).

(44)  
\[ a. \text{[apples]} = \lambda x_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{apple}(x) \]
\[ b. \text{[[M-OP_{2} apples]]}^{c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \lambda \alpha_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[apple}(\alpha) \land \mu_{2}(\alpha) \geq d]\]
\[ c. \text{[[d-M-OP_{2} apples]]}^{c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \lambda \alpha_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[apple}(\alpha) \land \mu_{2}(\alpha) \geq d]\]
\[ d. \text{[[\lambda d [[\text{[dp [Liubei]]} \cdot \text{bought \ [dp \ \exists \ \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[M-OP_{2} apples]]}] \ldots C]]^{w,c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[Liubei bought d or more apples}\}
\]
\[ e. C = \{ \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[y bought d or more apples}} \mid y \in D_{c}\} \]
\[ f. \text{[[(41)]]}^{w,c} = 1 \text{ iff } \forall y(y \in C \land y \neq \text{Liubei} \rightarrow \mu_{1}\{d: \text{Liubei bought d or more apples}\} \]
\[ > \mu_{1}\{d: \text{y bought d or more apples}\} \]

7 One way to incorporate the contribution of classifiers and kind terms is illustrated below (see Krifka 1995b, Chierchia 1998, Yang 2001, among others):

(i) a. \[ \text{[[pinguo]]}^{w} = \lambda z_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{atom}(z) \]
\[ \text{b. [[ke-pingo\ldots uo]]}^{w,c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[atom}(z) \land \mu_{2}(z) \geq d]\]
\[ \text{c. [[M-OP_{2} ke-pingo\ldots uo]]}^{w,c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[atom}(z) \land \mu_{2}(z) \geq d]\]
\[ \text{d. [[\lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[M-OP_{2} ke-pingo\ldots uo]]}^{w,c} = \lambda z_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[atom}(z) \land \mu_{2}(z) \geq d]\]
\[ \text{e. [[\lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[Liubei]} \cdot \text{bought [[dp \ \exists \ \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[M-OP_{2} apples]]} \ldots C]]}^{w,c} = \lambda d_{<\ldots, \ldots \ldots, \ldots >} \cdot \text{[bought (Liubei, z) \land \mu_{2}(z) \geq d]}\]
\[ \text{f. The sentence (81) is true iff } \]
\[ \forall y(y \in C \land y \neq \text{Liubei} \rightarrow \max\{d: \exists z\{\text{atom}(z) \land \text{bought (Liubei, z) \land } \mu_{2}(z) \geq d}\} \]
\[ > \max\{d: \exists z\{\text{atom}(z) \land \text{bought (y, z) \land } \mu_{1}(z) \geq d}\} \]

8 Given that quantity adjective *duo* ‘many/ much’ and M-OP both encode a measure function \(\mu\) and are evaluated relative to context, the number index 1 and 2 are used here to distinguish the two types of measurement. In particular, for the context \(c\) for QSs in the nominal domain like (82), \(g_{c}(2)\) is a measure function \(\mu_{2}\) that assigns cardinalities, while \(g_{c}(1)\) is a measure function \(\mu_{1}\) that measures interval sizes. As we will see shortly, for the context \(c\) for QSs in the verbal domain, \(g_{c}(2)\) is a measure function \(\mu_{2}\) that assigns a contextually-given dimension respecting monotonicity (e.g., temporal duration or distance), and \(g_{c}(1)\) is a measure function \(\mu_{1}\) that measures interval sizes.
(41) $w^c = 1 \text{ iff } \forall y[y \in C \land y \neq \text{Liubei} \rightarrow \\
\mu_1\{d: \exists\text{apple}(z) \land \text{bought}(\text{Liubei}, z) \land \mu_2(z) \geq d}\} \\
> \mu_1\{d: \exists\text{apple}(y) \land \text{bought}(y, z) \land \mu_2(z) \geq d}\}$

g. In words: for all $y$ mentioned in $C$, the number of apples Liubei bought exceeds the number of apples $y$ bought.

(44a) is given by the semantics of the noun pinguo ‘apples’. (44b) is given by functional application, by applying M-OP to (44a). (44c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (44d) is obtained by the lambda-abstraction, resulting from the movement of SupP. (44e) provides the contextual value of the domain $C$. (44f) is a consequence of functional application, by applying SupP (where the prejacent is of type $<d, t>$) to (44d). Finally, according to (44g), the sentence (40) is judged true if and only if the number of apples that Liubei bought is more than that of the apples that any other individual bought.

Crucially, exactly the same superlative component SupP (see (43)) is involved in both QSs and SMs. In particular, in the case of QSs, the prejacent is of type $<d, t>$ and the domain $C$ is in turn of type $<d, t, t>$. With these type-theoretical setting, the semantics of SupP is equivalent to the two-place superlative operator introduced in Heim (1999), as presented in (45). Thus, the morpho-semantic puzzle of SMs posed by Chinese zuiduo is captured.

\[
\begin{align*}
(45) \quad \text{a. } [-est] &= \lambda C.\lambda P_{<d, t>} \forall Q[Q \in C \land Q \neq P \rightarrow \max(\lambda d.P(d)) \\
&> \max(\lambda d.Q(d))] \\
\text{b. Presuppositions: } &P \in C, \exists Q[Q \in C \land Q \neq P]
\end{align*}
\]

Let’s consider the case of zuishao. The relevant sentence is given in (46), along with its LF in (47). Recall that the superlative expression zuishao is now structurally decomposed as the syntactic chunk [zui-Comp duo1]-C (section 3.2). As we have seen in the case of zuiduo as QSs, the superlative phrase SupP moves (thus creating a degree property) and scopes over the whole sentence.

(46) QSs in Nominal Domain

(Zhe-ci) Liubei mai-le zuishao (ke) pinguo.

This-time Liubei buy-ASP SUP-few CL apple

‘(This time,) Liubei bought fewer apples than anyone else did.’

(47) \[\text{SupP } \text{zui-Comp } \text{duo1}-\text{Cld } [\text{IP} \text{[IP [Liubei]f bought} \\
\text{[DP } \text{[AdjP } d-[[\text{M-OP}_2 \text{ apples}]]] \text{]} \text{]}]} \text{]} \text{]} \text{]} \text{]C}
\]

The semantics of SupP involved in zuishao is repeated in (48). Recall that in the case of QSs, $\alpha$ ranges over events and individuals, thus $\eta$ could be of type $<v>$ or $<e>$. As before, M-OP is generalized for both nominal domain and verbal domain.

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9 I am greatly indebted to Roger Schwarzschild for bringing my attention to the two-place superlative operator discussed in Heim (1999) and suggesting this line of analysis unifying Chinese SMs and QSs.
With these meaning pieces in place, the relevant computation is offered below. The computation of (47) is basically the same as the case of zuiduo, except for the opposite comparison relation: a less-than relation is encoded by Comp⁻ for zuishao; in contrast, a greater-than relation is encoded by Comp⁺ for zuishao.

(50) a. \[ \text{[apples]} = \lambda x_{<e}. \text{apple}(x) \]

b. \[ \text{[M-OP} \text{ apples]} = \lambda d_{<dp}. \lambda x_{<e}. \text{[apple}(x) \land \mu_2(a) \geq d] \]

c. \[ \text{[d-[M-OP} \text{ apples]} = \lambda x_{<e}. \text{[apple}(x) \land \mu_2(a) \geq d] \]

d. \[ \lambda d \text{ [[dp [Liubei] bought [dp [AdjP d-[M-OP} \text{ apples]]]]-C]]^{w,c} = \lambda d. \text{Liubei bought } d \text{ or more apples} \]
\[ = \lambda d. \exists z [ \text{apple}(z) \land \text{bought (Liubei, } z) \land \mu_2(z) \geq d] \]

e. \[ C = \{ \lambda d. y \text{ bought } d \text{ or more apples} \mid y \in D_e \} \]

f. \[ \text{[[47]]}^{w,c} = 1 \text{ iff } \forall y \in C \land y \neq \text{Liubei} \rightarrow \mu_1\{d : \text{Liubei bought } d \text{ or more apples}\} < \mu_1\{d : y \text{ bought } d \text{ or more apples}\} \]

\[ \text{[[47]]}^{w,c} = 1 \text{ iff } \forall y \in C \land y \neq \text{Liubei} \rightarrow \mu_1\{d : \exists z [ \text{apple}(z) \land \text{bought (Liubei, } z) \land \mu_2(z) \geq d]\} < \mu_1\{d : \exists z [ \text{apple}(z) \land \text{bought (y, } z) \land \mu_2(z) \geq d]\} \]

g. In words: for all y mentioned in C, the number of apples Liubei bought is less than the number of apples y bought.

(50a) is given by the semantics of the noun pinguo ‘apples’. (50b) is given by functional application, by applying M-OP to (50a). (50c) is obtained by the movement of the superlative phrase SupP, which leaves a degree variable saturating the degree argument of M-OP. (50d) is obtained by the lambda-abstraction, resulting from the movement of SupP. (50e) provides the contextual value of the domain C. (50f) is a consequence of functional application, by applying SupP (where the prejacent is of type \(<d, t>\)) to (50d). Finally, according to (50g), the sentence (46) is true if and only if the number of apples that Liubei bought is less than that of the apples that any other individual bought. Crucially, the same superlative component SupP (see (48)) is again involved in both QSs and SMs. The morpho-semantic puzzle posed by Chinese zuishao is thus captured.

4 Concluding remarks

Taking Chinese SMs as a case study, I have presented a decompositional analysis capturing three semantic properties of SMs: focus-sensitivity, the compatibility with various scales and their discrepancy, the bounding property. Crucially, the
analysis here cannot be the whole story for the morpho-semantic puzzle of SMs because there is more than one possible morpho-semantic mapping as shown below.

(51) Quantity adjectives plus even-if (e.g., Japanese and Korean)
   a. ooku-temo ‘at most’ Japanese
      many-even.if
   b. suknaku-temo ‘at least’
      few-even.if

(52) Quantity adjectives plus comparatives (e.g., Magahi, Hindi, Russian)
   a. jaadaa se aadaa ‘at most’ Magahi
      more than more
   b. kam se kam ‘at least’
      less than less

I’ve shown that the morphology of SMs is closely connected with their semantics. More studies are needed to see how the connection works in (51) and (52).

References